Time-Varying Parameter Vector Autoregressive (TVP-VAR) models are commonly used in econometrics to capture the dynamic relationships among multiple time series with parameters that can change over time. The concept of "connectedness" in this context typically refers to the degree of interdependence between different variables or financial assets over time, often applied to measure systemic risk in financial markets.

Here’s an overview of the key formulas and equations related to the **TVP-VAR connectedness** framework:

### **1. TVP-VAR Model**

A **TVP-VAR** model for KKK time series variables at time ttt can be written as:

Yt=ΦtYt−1+ϵtY\_t = \Phi\_t Y\_{t-1} + \epsilon\_tYt​=Φt​Yt−1​+ϵt​

Where:

* YtY\_tYt​ is a K×1K \times 1K×1 vector of endogenous variables (time series).
* Φt\Phi\_tΦt​ is a K×KK \times KK×K time-varying coefficient matrix that changes over time.
* ϵt\epsilon\_tϵt​ is a K×1K \times 1K×1 vector of error terms or innovations, assumed to follow a multivariate normal distribution with mean zero and time-varying covariance matrix Σt\Sigma\_tΣt​.

### **2. Time-Varying Parameter Evolution**

The parameters Φt\Phi\_tΦt​ in the TVP-VAR model follow a stochastic process, typically modeled as a random walk:

Φt=Φt−1+ηt\Phi\_t = \Phi\_{t-1} + \eta\_tΦt​=Φt−1​+ηt​

Where:

* ηt\eta\_tηt​ is a K×KK \times KK×K matrix of innovations associated with the time-varying parameters, assumed to be normally distributed.

### **3. Variance Decomposition (Forecast Error Variance Decomposition - FEVD)**

In connectedness analysis, one of the key metrics is the variance decomposition, which helps quantify how much of the forecast error variance of a particular variable can be attributed to shocks from other variables. The generalized forecast error variance decomposition (FEVD) is given by:

θij(H)(t)=σii−1∑h=0H−1(ei′Ah(t)Σtej)2∑h=0H−1(ei′Ah(t)ΣtAh′(t)ei)\theta\_{ij}^{(H)}(t) = \frac{\sigma\_{ii}^{-1} \sum\_{h=0}^{H-1} \left( e\_i' A\_h(t) \Sigma\_t e\_j \right)^2}{\sum\_{h=0}^{H-1} \left( e\_i' A\_h(t) \Sigma\_t A\_h'(t) e\_i \right)}θij(H)​(t)=∑h=0H−1​(ei′​Ah​(t)Σt​Ah′​(t)ei​)σii−1​∑h=0H−1​(ei′​Ah​(t)Σt​ej​)2​

Where:

* θij(H)(t)\theta\_{ij}^{(H)}(t)θij(H)​(t) represents the portion of the HHH-step-ahead forecast error variance of variable iii attributable to shocks to variable jjj at time ttt.
* Ah(t)A\_h(t)Ah​(t) is the time-varying coefficient matrix for the hhh-th lag in the moving average representation of the TVP-VAR.
* Σt\Sigma\_tΣt​ is the time-varying covariance matrix of the innovations ϵt\epsilon\_tϵt​.
* eie\_iei​ and eje\_jej​ are selection vectors, picking out the iii-th and jjj-th variables.
* HHH is the forecast horizon.

### **4. Connectedness Measures**

Several connectedness measures can be derived from the variance decomposition:

#### **a) Total Connectedness Index (TCI):**

This measure quantifies the overall degree of connectedness in the system:

TCIt=1K∑i=1K∑j=1,j≠iKθij(H)(t)TCI\_t = \frac{1}{K} \sum\_{i=1}^K \sum\_{j=1, j \neq i}^K \theta\_{ij}^{(H)}(t)TCIt​=K1​i=1∑K​j=1,j=i∑K​θij(H)​(t)

Where:

* TCItTCI\_tTCIt​ is the total connectedness at time ttt, representing the average percentage of forecast error variance due to shocks from other variables in the system.

#### **b) Directional Connectedness (From Others and To Others):**

The connectedness from variable iii to others and from others to iii can be calculated as:

* **From Others:**

Ci←∙(t)=∑j=1,j≠iKθij(H)(t)C\_{i \leftarrow \bullet}(t) = \sum\_{j=1, j \neq i}^K \theta\_{ij}^{(H)}(t)Ci←∙​(t)=j=1,j=i∑K​θij(H)​(t)

* **To Others:**

Ci→∙(t)=∑j=1,j≠iKθji(H)(t)C\_{i \rightarrow \bullet}(t) = \sum\_{j=1, j \neq i}^K \theta\_{ji}^{(H)}(t)Ci→∙​(t)=j=1,j=i∑K​θji(H)​(t)

#### **c) Net Pairwise Connectedness:**

The net connectedness between two variables iii and jjj is calculated as:

Cij(H)(t)=θij(H)(t)−θji(H)(t)C\_{ij}^{(H)}(t) = \theta\_{ij}^{(H)}(t) - \theta\_{ji}^{(H)}(t)Cij(H)​(t)=θij(H)​(t)−θji(H)​(t)

### **5. Dynamic Connectedness Measures**

In the TVP-VAR framework, connectedness measures change over time, reflecting how the interdependencies between variables evolve. These dynamic measures allow the study of systemic risk, contagion, and co-movements in financial markets.

The equations above can be used to analyze time-varying connectedness in the context of systemic risk, macroeconomic policy, and financial markets. They capture the essence of how shocks propagate through the system and help in understanding the interconnectedness in a dynamic framework.